Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Complex Analysis

Subject Code: 5SC01COA1 **Branch: M.Sc.(Mathematics)**

Semester: 1 **Date: 16/03/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Find the rectangular form of $z=\sqrt{2}\cos 510^{\circ}+\sqrt{2}\sin 510^{\circ}$. (02)

b) Find the zeros of $\cos z$. (02)

c) Define Entire function and give one example of it. (02)

d) True/False: $f(z)=z^2+2$ is analytic everywhere. (01)

Q-2 **Attempt all questions** **(14)**

a) Find all n^{th} root of unity and hence find the sum and product of all the roots. (06)

b) Prove that $\tan^{-1} z=\frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$ and hence find the value of $\tan^{-1}(2i)$. (06)

c) Find $\sqrt{-8+6i}$. (02)

OR

Q-2 **Attempt all questions** **(14)**

a) Define: $\log(x+iy)$ and find the real and imaginary part of $\left(\sqrt{i}\right)^{\sqrt{i}}$. (06)

b) Define: $\arg z$ and also find z if $\arg(z+2i)=\frac{\pi}{4}$ and $\arg(z-2i)=\frac{3\pi}{4}$. (06)

c) Find real and imaginary part of $\sinh z$. (02)

Q-3 **Attempt all questions** **(14)**

a) State and prove necessary and sufficient condition for function to be an analytic. (07)

b) If $f(z)=\begin{cases}\frac{(\bar{z})^2}{z}, & if\ z\neq 0\\ 0, & if\ z=0\end{cases}$ is not differentiable at the origin although Cauchy- (05)

Riemann equations are satisfied at origin.

c) Prove that every differentiable function is continuous function. (02)

OR

Q-3 **Attempt all questions** **(14)**

a) Suppose f is differentiable at z_0 and g is differentiable at $w_0 = f(z_0)$ then $g \circ f$ is differentiable at z_0 and $\left(g \circ f(z_0)\right)' = g'\left(f(z_0)\right) \cdot f'(z_0)$. (07)

b) Find the analytic function $f(z)$ if $u - v = (x-y)\left(x^2 + 4xy + y^2\right)$. (05)

c) Is $f(z) = \dfrac{z}{\bar{z}}$ an analytic? Explain it. (02)

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a) State Morera's theorem. (02)

b) Evaluate: $\oint_C \dfrac{1}{z^2 - 3z + 2}\,dz$, where $C: |z| = 1$ (02)

c) Find the residue of $f(z) = z^4 e^{\frac{1}{z}}$. (02)

d) Which are the fixed points of $w = \dfrac{2z-3}{z+2}$? (01)

Q-5 **Attempt all questions** **(14)**

a) Evaluate $\int_C f(z)\,dz$ where $f(z) = \begin{cases} 4y, & if\ y > 0 \\ 1\ , & if\ y < 0 \end{cases}$ and C is the arc from $z = -1 - i$ to $z = 1 + i$ of the cubical curve $y = x^3$. (05)

b) State and prove Cauchy's integral formula. (05)

c) Integrate the function $f(z) = \dfrac{z+4}{z^2 + 2z + 5}$ around the curve $C: |z| = 3$ traversed in counter-clockwise direction. (04)

OR

Q-5 **Attempt all questions** **(14)**

a) Evaluate $\oint_C \dfrac{z^4}{(z+1)(z-i)^2}\,dz;\ C$ is the ellipse $9x^2 + 4y^2 = 36$ by using Cauchy's integral formula. (05)

b) Let $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n\ (a_n \neq 0)$ be a complex valued polynomial of degree $n(n \geq 1)$ then there exist at least one complex root z_0 such that $P(z_0) = 0$. (05)

c) Find an upper bound for the absolute value of the integral $\int_C \dfrac{e^z}{z^2}\,dz$ where C is the line segment i to 2. (04)

Q-6 **Attempt all questions** **(14)**

a) Find all Taylor's and Laurent's series expansions for the function (05)

$f(z)=\dfrac{7z2+9z-18}{z^3-9z}$ about the point $z=0$.

b) Evaluate $\oint_C \dfrac{2z+1}{(z+1)^3(z-1)}dz;\ \ C:|z-1|=2.5$ by using Cauchy's residue theorem. (05)

c) Find the bilinear transformation which maps $0,1,i$ onto $1+i,-i,2-i$ respectively and also find the image of $|z|<1$. (04)

OR

Q-6 **Attempt all Questions** **(14)**

a) State and prove Taylor's theorem. (05)

b) Evaluate: $\displaystyle\int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+1)^3}$ (05)

c) Evaluate: $\displaystyle\int_{0}^{2\pi}\frac{\cos\theta}{5+4\cos\theta}d\theta$ (04)